IADU

Institute for Advanced Dynamic Uncertainty

Adversarial Uncertainty

The IADU Research Agenda Across Five Divisions

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What is adversarial uncertainty?

Most stochastic models assume noise is **passive** — a fair coin, a Brownian motion, a Poisson stream. Decisions are made against an environment that does not push back. **Adversarial uncertainty** is the opposite: the environment is allowed to be the worst possible, within constraints we control.

- A market against a risk manager
- A speculator against a central bank
- An emitter against a climate planner
- A counter-party against a portfolio
- A bad draw against a robust forecast

Every IADU division studies a different facet of this single idea.

A working definition

Given a state process X, a controller's action u, and an adversary's action v, we study the **value functional**

$$V(x) = \sup_{u \in \mathscr{U}} \inf_{v \in \mathscr{V}} \mathbb{E}_x \left[\int_0^\infty e^{-qt} \, \ell(X_t^{u,v}, u_t, v_t) \, dt \right]. \tag{1}$$

The controller maximises a worst-case payoff. The order of sup-inf matters, and when it doesn't (the **Isaacs condition**) the game has a clean saddle solution.
This single object reappears, in different forms, across every division of the institute.

The five divisions

Code	Division	Adversary takes the form of...
SA	Stochastic Analysis and Control	A worst-case operator or measure
GS	Games, Dynamics and Strategic Control	A second player with conflicting payoff
FM	Financial Mathematics and Asset Pricing	A pricing measure or hedging counter-party
PM	Quantitative Policy and Macroeconomics	A constrained-rational agent against a planner
EN	Sustainability and Energy Economics	An emitter or consumer against a regulator

Each division has its own toolkit — but the mathematical objects (HJB, HJI, resolvents, Itô-Lévy) are shared.

Question: What is the right operator-theoretic framework when the noise driving a state has jumps?
Tools: resolvent operators, Wiener-Hopf factorisation, viscosity solutions, Itô-Lévy calculus.
Headline result of the division (2023): the resolvent of a one-dimensional Lévy process admits a unified factorisation valid for the full Kuznetsov Λ-meromorphic class — recovering Kou, hyperexponential, and CGMY families as special cases.
This operator register is the **lingua franca** the other divisions cite.

Question: When the state evolves under a worst-case adversary, what replaces the Bellman equation?

Tools: Hamilton-Jacobi-Isaacs (HJI) integro-differential equations, viscosity comparison principles, non-local first-order conditions for saddle pairs.

Recent result (2023): for two-player zero-sum SDGs driven by Lévy noise, value functions are $1/2$-Hölder and coincide as the viscosity solution of an integro-differential HJI under the Isaacs condition.

A worked example — a central-bank-versus-speculator drift-and-intensity game — admits a closed-form saddle via the Wiener-Hopf root.

Division FM — Financial Mathematics & Asset Pricing

Question: What does worst-case mean when the adversary is the market itself — and the controller is a hedger or risk manager?
Tools: risk-neutral pricing under model ambiguity, robust utility maximisation, G-expectation, BSDEs under uncertainty.
Recent direction: implied-volatility-surface calibration under jump noise, with no-arbitrage constraints enforced as convex feasibility — every recalibration is a small quadratic program, solved in milliseconds.
The link to GS is direct: a market-maker against an informed trader is a two-player zero-sum game on a price process with jumps.

Division PM — Quantitative Policy & Macroeconomics

Question: A policy maker sets rules; private agents respond. If the agents act adversarially (in a Knightian sense), what is the right macro model?
Tools: continuous-time mean-field games, heterogeneous-agent HJB-FP systems, robust optimal taxation under model ambiguity.
Active line: HJB-based estimators for output-gap uncertainty — a central bank chooses a tax-rule trajectory against a constrained-rational household sector whose preferences are not perfectly known.

Division EN — Sustainability & Energy Economics

Question: A regulator picks a carbon price path; emitters optimise against it; an adversarial-shock model handles deep climate uncertainty.
Tools: finite-horizon HJB for optimal carbon taxes, integrated-assessment models with Knightian shocks, real-options analysis of green investment under jump risk.
Headline result (2026): the optimal carbon-tax trajectory under a Kou-jump climate-shock process is monotone in the up-jump intensity and admits a closed-form steady state when the social-cost-of-carbon function is linear-quadratic.

The shared mathematical core

All five divisions reduce, eventually, to one of three objects:

$$\textbf{HJB:} \quad qV = \sup_u \big[H(x, \partial V, \partial^2 V, u) + \mathscr{I}[V](x) \big] \tag{2}$$

$$\textbf{HJI:} \quad qV = \sup_u \inf_v \big[H(x, \partial V, \partial^2 V, u, v) + \mathscr{I}[V](x; v) \big] \tag{3}$$

$$\textbf{Resolvent:} \quad \mathscr{R}_q f(x) = \mathbb{E}_x \left[\int_0^\infty e^{-qt} f(X_t)\, dt \right] = (q - \mathscr{L})^{-1} f(x) \tag{4}$$

The institute's research is what happens when these three meet adversarial uncertainty.

Cross-division collaboration patterns

The institute's most-cited results come from work that crosses division lines:

- **SA × GS** — operator-theoretic comparison principles for HJI integro-differential equations
- **GS × FM** — strategic price-impact models as two-player games on a Lévy state
- **PM × EN** — robust carbon-tax policy as a continuous-time MFG under Knightian shocks
- **SA × FM** — implied-vol surface calibration via Wiener-Hopf roots of the price-process resolvent
- **GS × PM** — robust monetary policy as a central-bank-versus-market game

Every line above corresponds to at least one IADU research paper.

Methods we use in practice

A typical IADU paper combines three of the following:

- **Itô–Lévy calculus** to handle jumps in the state process
- **Viscosity solutions** to handle non-smooth value functions
- **Wiener–Hopf factorisation** to close the linear part of the HJB/HJI
- **Howard policy iteration** to compute the saddle numerically
- **Comparison principles** to prove uniqueness across viscosity classes
- **BSDE / G-expectation** when the adversarial measure is the unknown

The choice is driven by what the adversary controls, not by which division writes the paper.

A worked illustration

The 2023 paper *Saddle-Point Structures for Two-Player Zero-Sum SDEs with Lévy Noise* is a complete instance of the institute's research style:

$$qV^{\pm}(x) = \sup_{u} \inf_{v} \big\{ H(x, \partial_x V^{\pm}, \partial_{xx} V^{\pm}, u, v) + \mathscr{I}[V^{\pm}](x; v) \big\}. \tag{5}$$

Under the Isaacs condition, $V^{+} = V^{-} = V$. The saddle satisfies a **non-local** first-order condition because the Lévy integral survives differentiation. The bank-versus-speculator example admits a closed-form saddle via the resolvent's Wiener-Hopf root from the SA division.

One paper, three divisions' methods, one adversarial frame.

Open problems

The institute's current research register has four open lines:

- **Multi-player games on Lévy states** — Isaacs becomes a vector condition; uniqueness is open
- **Mean-field games with Knightian shocks** — coupled HJB-FP-jump system, no general well-posedness theorem
- **Robust calibration under jump-clustering noise** — Hawkes-type intensities break Wiener-Hopf separability
- **Climate-policy under deep uncertainty** — no consensus social-cost-of-carbon functional under Knightian ambiguity

Each is open enough to be a fellowship project; concrete enough to be a paper.

How to engage with IADU

Three channels, in increasing order of commitment:

- **Read the research register** at ‘iadu.org/research/‘ — every paper has a downloadable PDF and a one-line summary card
- **Attend presentations** at ‘iadu.org/presentations/‘ — conference talks, keynotes, workshops, all paginated with the same standards as the papers
- **Collaboration** — write to ‘research@iadu.org‘ with a one-page sketch of a joint project; the institute funds visiting researchers on a rolling basis

Thank you — questions welcome

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